Correction to "Using the pseudo-dimension to analyze approximation algorithms for integer programming"

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Abstract. This note corrects a mistake in the WADS'01 paper titled "Using the pseudo-dimension to analyze approximation algorithms for integer programming".

Section 3 of [1] is about integer programs of the form

$$\min c^T x, \text{ s.t. } Ax \geq b \text{ and } x \geq (0, 0, ..., 0)^T$$

where all components of b and c, and all entries of A, are non-negative. It includes the claim that, without further loss of generality, all components of c are in fact strictly positive. This is correct: if any $c_j = 0$, whenever $A_{ij} > 0$ the ith constraint can be satisfied at zero cost by using a large value of x_j, so we arrive at an equivalent problem by removing x_j and all such constraints.

Section 4 is about the class of integer programs obtained from the above by allowing negative entries in A. It includes the claim that, also in this case, w.l.o.g. all components of c are strictly positive, for the same reason. This is *not* correct. A counterexample is

$$\min x_1, \text{ s.t. } x_1 - x_2 \geq 0, x_1 \geq 0, x_2 \geq 1.$$

The optimum is 1, and, if we remove x_2 and the constraint $x_2 \geq 1$, the optimum becomes 0.

Theorem 2 from Section 4 is applied to the minimum majority problem in Section 6.2. For this problem, all components of c are strictly positive.

Fortunately, it appears that the mistake in Section 4 has not misled anyone to try to apply Theorem 2 to show that 3-colorable graphs can be colored with $O(\log n)$ colors in polynomial time.

References

1. Philip M Long. Using the pseudo-dimension to analyze approximation algorithms for integer programming. In *Workshop on Algorithms and Data Structures*, pages 26–37. Springer, 2001.